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MAM1020S **Tutorial 8** **August 2017**

1. Find the following limits. You may use L'Hopital's rule where appropriate. Be aware that L'Hopital's rule may not apply to every limit, and it may not be helpful even when it does apply. Some limits may be found by other methods. Prefer one of the methods seen before the hospital rule if it can apply. These problems are given in no particular order.

1. $lim_{x\to 0}\frac{e^x-x-1}{cos(x)-1}$;
2. $lim_{x\to -2}\frac{x^3-x^2-10x-8}{5x^3+12x^2-2x-12}$;
3. $lim_{x\to a}\frac{x-a}{ln(x)-ln(a)}$, where $a>0$;
4. $lim_{\theta\to 0}\frac{1-cos(\theta)}{\theta^2}$;
5. $lim_{x\to\infty}(sinh(x)-cosh(x))$
6. $lim_{x\to\pi/2^+}\frac{tan(x)}{ln(2x-\pi)}$
7. $lim_{x\to\infty}\frac{tanh(x)}{tan^{-1}(x)}$;
8. $lim_{x\to 0}\frac{sin(x)}{sinh(x)}$;
9. $lim_{x\to 0}\left(\frac{e^x}{e^x-1}-\frac{1}{x}\right)$;
10. $lim_{x\to\pi/2^+}\frac{ln(x-\pi/2)}{sec(x)}$
11. $lim_{x\to 2}\frac{e^{x^2}-e^4}{x-2}$;
12. $lim_{x\to\infty}\frac{\sqrt[3]{x}}{ln(x)}$;
13. $lim_{x\to\pi/4}\frac{ln(tan(x))}{sin(x)-cos(x)}$
14. $lim_{x\to\infty}\frac{e^{-x}}{sin(x)+2}$
15. $lim_{x\to 1}\frac{tan^{-1}(x)-\pi/4}{tan(\pi/4x)-1}$
16. $lim_{x\to 0^+}sin(x)ln(x)$
17. $lim_{x\to 0}xsin(1/x)$;
18. $lim_{x\to\pi/2}\frac{tan(3x)}{tan(5x)}$;
19. $lim_{x\to\pi/2^-}\frac{sec(x)}{ln(sec(x))}$
20. $lim_{x\to 0}(csc(x)-1/x)$
21. $lim_{x\to\pi/2^-}(tan(x))^{sin(2x)}$
22. $lim_{x\to-\infty}x^{-3}e^x$
23. $lim_{x\to\infty}\frac{ln(t+2)}{log_2(t)}$
24. $lim_{x\to e}\frac{1-ln(x)}{x/e-1}$
25. $lim_{x\to\pi}\frac{\sqrt{1-tan(x)}-\sqrt{1+tan(x)}}{sin(2x)}$
26. $lim_{x\to\infty}\frac{\pi/2-tan^{-1}(x)}{x^{-1}}$
27. $lim_{x\to -1}\frac{\sqrt{x+10}+3x^{1/3}}{4x^2+3x-1}$
28. $lim_{x\to 0}\frac{sin(x)-x}{x^3}$
29. $lim_{x\to 0}\frac{tan(x)-x}{x^3}$
30. $lim_{x\to 2}\frac{x^4-4^x}{sin(\pi x)}$
31. $lim_{x\to 3\pi}\frac{1+tan(x/4)}{cos(x/2)}$
32. $lim_{x\to\infty}x^{1/ln(x)}$
33. $lim_{x\to 0}cos(x)^{csc(x)}$
34. $lim_{x\to\pi/2^-}cos(x)ln(tan(x))$
35. $lim_{x\to\infty}x^{sin(1/x)}$
36. $lim_{x\to\pi/2}(\pi/2-x)tan(x)$
37. $lim_{x\to 0^+}\frac{sin(3x)cot(2x)}{ln(cos(x))}$
38. $lim_{x\to 0}(1/x^2-cot(x)/x)$
39. $lim_{x\to 0}\frac{sin^{-1}(x)}{x^2csc(x)}$
40. $lim_{x\to 1}tan(\pi x/2)ln(x)$
41. $lim_{x\to -2}\frac{xe^x-4+2e^x-2x}{1+xsin(\pi x)+x/2+2sin(\pi x)}$
42. $lim_{x\to 1^-}\sqrt{1-x}ln(ln(1/x))$
43. $lim_{x\to 0^+}(csc(x))^{cos^{-1}(x)/ln(x)}$
44. $lim_{\theta\to 0}\frac{2sin(\theta)-sin(2\theta)}{sin(\theta)-\theta cos(\theta)}$
45. $lim_{x\to\infty}\frac{e^x+x}{sinh(x)}$
46. $lim_{\theta\to\pi/2}\frac{2(e^{cos(\theta)}+\theta-1)-\pi}{ln(sin(-3\theta))}$
47. $lim_{x\to 0^+}\frac{xsin(1/x)}{ln(x)}$
48. $lim_{x\to 0^+}\frac{xcot(x)}{e^x-1}$
49. $lim_{x\to 0^+}\frac{xln(x)}{ln(1+ax)}$, $a>0$.
50. $lim_{x\to 0}\frac{sin^{-1}(x)}{xcos^{-1}(x)}$
51. $lim_{x\to 0^+}\frac{ln(x)}{cot(x)}$
52. $lim_{x\to\infty}(ax)^{b/(cx)}$, $a,c\neq 0$
53. $lim_{x\to\infty}\frac{x^{10^{10}}}{e^x}$
54. $lim_{x\to 0}\frac{x^2cos(x)}{2sin^2(1/2x)}$
55. $lim_{x\to a}\frac{\sqrt{2a^3x-x^4}-a\sqrt[3]{a^2x}}{a-\sqrt[4]{ax^3}}$.

2. Find the value of

1. $tan(sin^{-1}(1/5))$
2. $sin(cos^{-1}(-3/5))$.
3. $sin(2arctan(-4/3))$
4. $arcsin(sin(\pi/8))$
5. $arccos(sin(\pi/8))$
6. $cos(arcsin(1/3))$

3. Simplify

   1. $sin(arctan(x))$
   2. $tan(arctan(x))$
   3. $cot(arctan(x))$
   4. $sec(arctan(x))$
   5. $arccos(sin(\theta))$, assuming that θ is in the interval $[0, \pi/2]$
   6. $arccos(y) + arcsin(y)$
   7. $cos(arcsin(1/3))$

4. Find the derivative of the following functions

   1. $f(x) = cos^{-1}(x^3)$ for $|x^3| < 1$.
   2. $f(x) = tan^{-1}(\sqrt{3x})$
   3. $f(t) = tan^{-1}t^4$
   4. $f(t) = tcot^{-1}(1 + t^2)$
   5. $f(t) = sin^{-1}(\sqrt{1 - t^4})$
   6. $f(t) = \frac{1}{\sqrt{1-t^2}} + cos^{-1}(t)$
   7. $f(x) = sin^{-1}(\sqrt{x})$
   8. $f(y) = cot^{-1}(\frac{y}{1-y^2})$

5. Find the a domain and codomain so that the following function are invertible and compute their inverse.

   1. $f(x) = sin(\sqrt{x}) + 2$.
   2. $f(x) = sin(\sqrt{x} + 2)$.
   3. $f(x) = sin(\sqrt{x + 2})$.
   4. $f(x) = e^{sin(\sqrt{x}+2)}$.
   5. $f(x) = sin(arccosx)$.

The final exam is around the corner, you finished all the exercise above here a selection of random exercises for final/test practice. Any of those could be part of the final/test and the rest of the tutorial

1. Find the derivative of the following, showing your work:
   (a) $f(x) = 4^{xsin(x)}$
   (b) $g(x) = ln|3x^2 - 4|$
   (c) $h(x) = sinh(\sqrt{tan(x)})$
   (d) $k(x) = x^{sec(x)}$
   (e) $f(x) = \frac{10^x}{x^2+1/x}$
   (f) $g(x) = e^x\sqrt{sin(x)}$
   (g) $h(x) = tan(\frac{e^{3x}}{x^3-3x^2})$
   (h) $f(x) = e^{xtan(x)}$
   (i) $g(x) = sin^4(\frac{ln(x)}{\sqrt{x}}$

2. Find the equation of the tangent line to the curve given by the parametric equation

$$x(t) = \sqrt{2}cos(t),\ y(t) = 2tan(t)$$

3. Let the function f be defined by

$$f(x) = \begin{cases} ax^2 & for\ x < -1 \\ x - b & for\ x \geq -1 \end{cases}$$

   (a) How should a and b be related if we want f to be continuous at $x = -1$? Give full reason for your answer.

(b) What should a and b be equal to if we want f to be differentiable at $x = -1$? Use the definition of $f'(-1)$ to obtain your answer.

4. Suppose f is differentiable at a. It follows that $lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ exists; let it be m. Let $E(x) = \frac{f(x)-f(a)}{x-a} - m$ for $x \neq a$. Then $f(x) = f(a) + (x-a)m + (x-a)E(x)$.

   (a) What do we usually call m?

   (b) What is $lim_{x \to a} E(x)$?

   (c) Use that $f(x) = f(a) + (x-a)m + (x-a)E(x)$ and show that f is continuous at a.

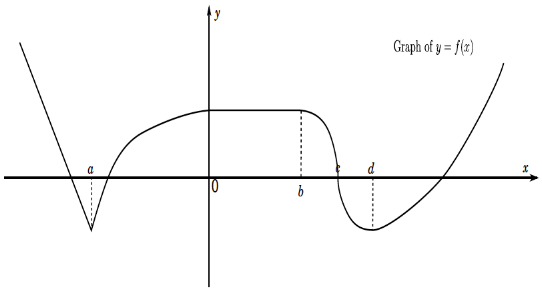
5. Find $lim_{x \to 0} \frac{\sqrt{1+tan(x)} - \sqrt{1+tan(x)}}{x^3}$ and show your working. (You may not use the Hospital rule!)

6. You are given that the equation

$$x^2 - y^2 + 2xy = e^x y^5 - 2$$

   defines y as a function of x. Find dy/dx at $(0, 1)$.

7. Below is a sketch of the graph of $y = f(x)$



   Draw a rough sketch of the graph of $y = f'(x)$. Be careful to indicate where f' is not defined.

8. Let $f(x) = x^{2/3}(x-4)^2$

   (a) Find $f'(x)$; show your working.

   (b) Factorize your previous answer and show that

$$f'(x) = \frac{8(x-4)(x-1)}{3x^{1/3}}$$

   (c) Where does f have horizontal tangent? Where is $f' > 0$? Where is f' undefined?

   (d) Sketch a rough graph of f using the above. (You should not consider y'' or concavity)

9. Let the function f be defined by

$$f(x) = \begin{cases} ax^2 + bx + 1 & for\ x < 1 \\ 2x + 3 & for\ x \geq 1 \end{cases}$$

   (a) How should you choose a and b if we want f to be continuous at $x = 1$? Give full reason for your answer.

   (b) What should a and b be equal to if we want f to be differentiable at $x = 1$? Use the definition of $f'(1)$ to obtain your answer.

10. Find $lim_{h \to 0} \frac{1-cos(h)}{h^2}$. Show your working. (You may use the usual rules for limits and the known limit $lim_{h \to 0} \frac{sin(h)}{h}$

11. A continuous function f that is defined for all x has the following properties: $f' > 0$, $f'' < 0$, $f(5) = 2$ and $f'(5) = 1$.

   (a) Find the equation of the tangent to f at $x = 5$.

(b) Give a possible sketch of f and include a sketch of the tangent you found in the previous question.

(c) Could f have an x-intercept that is negative? Explain your answer.

(d) Is it possible that $f'(1) = 1/2$? Explain you answer.

12. Suppose that f is differentiable and that it is also increasing (for all x). Is $(f(x))^3$ increasing for all x? If not give a counter example that shows this. If it is increasing, give a proof that it is increasing.

13. Consider the statement:
If $(f(x))^2$ is differentiable for all x then $f(x)$ is differentiable for all x. If this true or false. Give reasons for your answer.

14. Assume that the following equation defines y as a function of x

$$x^3 - 2xy + y^2 = 1.$$

Find dy/dx and find the equation of the tangent to the curve at $(1, 2)$.

15. Solve the inequality $|x - 1| < x + 2$ (show your working).

16. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a 1-1 function. Does it follow that the function g defined by $g(x) = f(x - 1)$ is 1-1? If your answer is yes explain why. If your answer is no give a counter example.

17. Use the definition of the derivative to show that if f is even and differentiable for all x then f' is odd.

18. Suppose that $f : [0, 1] \to [0, 1]$ and $g : [0, 1] \to [0, 1]$ are functions such that $g \circ f$ is $1 - 1$ Does it follows that g is $1 - 1$. If your answer is "yes" explain why. If it is "No" give a counter example.